10 Apêndice

10.1 Constantes Físicas - 1

$k = 1{,}381 \cdot 10^{-23}\ J/K$	$k = 8{,}617 \cdot 10^{-5}\ eV/K$
$N_A = 6{,}023 \cdot 10^{23}\ /mol$	$e = 1{,}602 \cdot 10^{-19}\ C$
$m_e = 9{,}109 \cdot 10^{-31}\ kg$	$m_e = 0{,}5110\ MeV/c^2$
$m_p = 1{,}672 \cdot 10^{-27}\ kg$	$m_p = 938{,}3\ MeV/c^2$
$m_n = 1{,}675 \cdot 10^{-27}\ kg$	$m_n = 939{,}6\ MeV/c^2$
$h = 6{,}626 \cdot 10^{-34}\ Js$	$h = 4{,}136 \cdot 10^{-15}\ eVs$
$\hbar = 1{,}055 \cdot 10^{-34}\ Js$	$\hbar = 0{,}6583 \cdot 10^{-15}\ eVs$
$hc = 12{,}41 \cdot 10^{-7}\ eVm$	$\hbar c = 1{,}975 \cdot 10^{-7}\ eVm$
$1/4\pi\epsilon_o = 8{,}988 \cdot 10^{9}\ Jm/C^2$	$c = 2{,}998 \cdot 10^{8}\ m/s$
$1eV = 1{,}602 \cdot 10^{-19}\ J$	$\sigma = 5{,}67 \cdot 10^{-8}\ W/m^2K^4$

10.2 Constantes Físicas -2

Velocidade da luz no vácuo	$c = 3{,}00{\times}10^{8}$ m/s
Constante de Planck	$h = 6{,}63{\times}10^{-34}$ J s $= 4{,}14{\times}10^{-15}$ eV s
	$hc = 2{,}00{\times}10^{-25}$ J m $= 1{,}24{\times}10^{-6}$ eV m
Constante magnética	$\mu_0 = 4\pi{\times}10^{-7}$ N/A^2 $= 12{,}6{\times}10^{-7}$ N/A^2
Constante elétrica	$\epsilon_0 = \dfrac{1}{\mu_0 c^2} = 8{,}85{\times}10^{-12}$ F/m
	$\dfrac{1}{4\pi\epsilon_0} = 8{,}99{\times}10^{9}$ m/F
Constante gravitacional	$G = 6{,}67{\times}10^{-11}$ N m^2/kg^2
Carga elementar	$e = 1{,}60{\times}10^{-19}$ C
	1 eV $= 1{,}60{\times}10^{-19}$ J
Massa do elétron	$m_{\rm e} = 9{,}11{\times}10^{-31}$ kg $= 511$ keV/c^2
Comprimento de onda Compton	$\lambda_{\rm C} = 2{,}43{\times}10^{-12}$ m
Massa do próton	$m_{\rm p} = 1{,}673{\times}10^{-27}$ kg $= 938$ MeV/c^2
Massa do nêutron	$m_{\rm n} = 1{,}675{\times}10^{-27}$ kg $= 940$ MeV/c^2
Massa do dêuteron	$m_{\rm d} = 3{,}344{\times}10^{-27}$ kg $= 1.876$ MeV/c^2
Massa da partícula α	$m_\alpha = 6{,}645{\times}10^{-27}$ kg $= 3.727$ MeV/c^2
Constante de Rydberg	$R_\infty = 1{,}10{\times}10^{7}$ m^{-1}
	$R_\infty hc = 13{,}6$ eV
Raio de Bohr	$a_0 = 5{,}29{\times}10^{-11}$ m
Constante de Avogadro	$N_{\rm A} = 6{,}02{\times}10^{23}$ mol^{-1}
Constante de Boltzmann	$k_{\rm B} = 1{,}38{\times}10^{-23}$ J/K
Constante molar dos gases	$R = 8{,}31$ J mol^{-1} K^{-1}
Constante de Stefan-Boltzmann	$\sigma = 5{,}67{\times}10^{-8}$ W m^{-2} K^{-4}

Raio do Sol	$= 6{,}96{\times}10^{8}$ m	Massa do Sol	$= 1{,}99{\times}10^{30}$ kg
Raio da Terra	$= 6{,}37{\times}10^{6}$ m	Massa da Terra	$= 5{,}98{\times}10^{24}$ kg
Distância Sol-Terra	$= 1{,}496{\times}10^{11}$ m		

10.3 Constantes numéricas

$\pi \cong 3{,}142$	$\ln 2 \cong 0{,}693$	$\cos(30°) = \sqrt{3}/2 \cong 0{,}866$
$\mathrm{e} \cong 2{,}718$	$\ln 3 \cong 1{,}099$	$\mathrm{sen}(30°) = 1/2$
$1/\mathrm{e} \cong 0{,}368$	$\ln 5 \cong 1{,}609$	
$\log_{10} \mathrm{e} \cong 0{,}434$	$\ln 10 \cong 2{,}303$	

10.4 Eletromagnetismo

10.4.1 Equações de Maxwell

$$\oint \mathbf{E}\cdot d\vec{\ell} + \frac{\partial}{\partial t}\int \mathbf{B}\cdot d\mathbf{S} = 0 \qquad \nabla\times\mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\oint \mathbf{B}\cdot d\mathbf{S} = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

$$\oint \mathbf{D}\cdot d\mathbf{S} = Q = \int \rho dV \qquad \nabla\cdot\mathbf{D} = \rho$$

$$\oint \mathbf{H}\cdot d\vec{\ell} - \frac{d}{dt}\int \mathbf{D}\cdot d\mathbf{S} = I = \int \mathbf{J}\cdot d\mathbf{S} \qquad \nabla\times\mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$$

10.4.2 Outras relações

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon\mathbf{E} \qquad \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu\mathbf{H}$$

$$\oint \mathbf{P}\cdot d\mathbf{S} = -Q_P \qquad \nabla\cdot\mathbf{P} = -\rho_P \qquad \oint \mathbf{M}\cdot d\vec{\ell} = I_M \qquad \nabla\times\mathbf{M} = \mathbf{J}_M$$

$$V = -\int \mathbf{E}\cdot d\vec{\ell} \qquad \mathbf{E} = -\nabla V \qquad d\mathbf{H} = \frac{I d\vec{\ell}\times\hat{\mathrm{e}}_r}{4\pi r^2}$$

$$\mathbf{B} = \nabla\times\mathbf{A} \qquad \mathbf{A} = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}dV}{r}$$

$$d\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}\hat{\mathrm{e}}_r \qquad dV = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r} \qquad \mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \qquad d\mathbf{F} = I d\vec{\ell}\times\mathbf{B}$$

$$\mathbf{J} = \sigma\mathbf{E} \qquad \nabla\cdot\mathbf{J} + \frac{\partial\rho}{\partial t} = 0$$

$$u = \frac{1}{2}(\mathbf{D}\cdot\mathbf{E} + \mathbf{B}\cdot\mathbf{H}) = \frac{\epsilon}{2}E^2 + \frac{1}{2\mu}B^2 \qquad \mathbf{S} = \mathbf{E}\times\mathbf{H}$$

$$(\rho = 0,\ \mathbf{J} = \mathbf{0}) \Rightarrow \nabla^2\mathbf{E} = \mu\epsilon\frac{\partial^2\mathbf{E}}{\partial t^2}$$

$$U_C = \frac{1}{2}\frac{Q^2}{C} \qquad U_L = \frac{1}{2}LI^2$$

10.5 Relatividade

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \qquad x' = \gamma\,(x - Vt) \qquad v'_x = \frac{v_x - V}{1 - V v_x/c^2}$$

$$t' = \gamma\left(t - Vx/c^2\right) \qquad y' = y \qquad v'_y = \frac{v_y}{\gamma\,(1 - V v_x/c^2)}$$

$$z' = z \qquad v'_z = \frac{v_z}{\gamma\,(1 - V v_x/c^2)}$$

$$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel} \qquad \mathbf{E}'_{\perp} = \gamma\,(\mathbf{E}_{\perp} + \mathbf{V} \times \mathbf{B})$$

$$\mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel} \qquad \mathbf{B}'_{\perp} = \gamma\left(\mathbf{B}_{\perp} - \frac{\mathbf{V}}{c^2} \times \mathbf{E}\right)$$

$$E = mc^2 = \gamma m_0 c^2 = m_0 c^2 + K \qquad E = \sqrt{(pc)^2 + (m_0 c^2)^2}$$

10.6 Resultados matemáticos úteis

$$\int_{-\infty}^{\infty} x^{2n} \mathrm{e}^{-\alpha x^2} dx = \frac{1.3.5...(2n{+}1)}{(2n{+}1)2^n \alpha^n} \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}} \qquad \sum_{k=0}^{\infty} q^k = 1/(1-q), \quad (q < 1)$$

$$\int \frac{\mathrm{d}u}{u(u-1)} = \ln(1 - 1/u) \qquad \mathrm{e}^{i\theta} = \cos\theta + i\,\mathrm{sen}\,\theta$$

$$\int \frac{dz}{(a^2 + z^2)^{1/2}} = \ln\left(z + \sqrt{z^2 + a^2}\right) \qquad \cos\theta = \frac{\mathrm{e}^{i\theta} + \mathrm{e}^{-i\theta}}{2}$$

$$\mathrm{sen}\,\theta = \frac{\mathrm{e}^{i\theta} - \mathrm{e}^{-i\theta}}{2i}$$

10.7 Harmônicos Esféricos

$$Y_{0,0} = \sqrt{\frac{1}{4\pi}} \qquad Y_{1,0} = \sqrt{\frac{3}{4\pi}} \cos\theta \qquad Y_{1,\pm 1} = \mp\sqrt{\frac{3}{8\pi}} \sin\theta \mathrm{e}^{\pm i\phi}$$

$$Y_{2,0} = \sqrt{\frac{5}{16\pi}} \left(3\cos^2\theta - 1\right) \qquad Y_{2,\pm 1} = \mp\sqrt{\frac{15}{8\pi}} \sin\theta\cos\theta \mathrm{e}^{\pm i\phi} \qquad Y_{2,\pm 2} = \mp\sqrt{\frac{15}{32\pi}} \sin^2\theta \mathrm{e}^{\pm 2i\phi}$$

10.8 Cálculo Vetorial

10.8.1 Coordenadas cartesianas

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathrm{e}}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathrm{e}}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathrm{e}}_z$$

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathrm{e}}_x + \frac{\partial f}{\partial y}\hat{\mathrm{e}}_y + \frac{\partial f}{\partial z}\hat{\mathrm{e}}_z \qquad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

10.8.2 Coordenadas cilíndricas

$$\nabla\cdot\mathbf{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \left[\frac{1}{r}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right]\hat{\mathrm{e}}_r + \left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right]\hat{\mathrm{e}}_\varphi + \left[\frac{1}{r}\frac{\partial(rA_\varphi)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \varphi}\right]\hat{\mathrm{e}}_z$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathrm{e}}_r + \frac{1}{r}\frac{\partial f}{\partial \varphi}\hat{\mathrm{e}}_\varphi + \frac{\partial f}{\partial z}\hat{\mathrm{e}}_z \qquad \nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

10.8.3 Coordenadas esféricas

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial(\mathrm{sen}\,\theta A_\theta)}{\partial \theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial(A_\varphi)}{\partial \varphi}$$

$$\nabla\times\mathbf{A} = \left[\frac{1}{r\sin\theta}\frac{\partial(\mathrm{sen}\,\theta A_\varphi)}{\partial \theta} - \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial A_\theta}{\partial \varphi}\right]\hat{\mathrm{e}}_r$$
$$+ \left[\frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial A_r}{\partial \varphi} - \frac{1}{r}\frac{\partial(rA_\varphi)}{\partial r}\right]\hat{\mathrm{e}}_\theta + \left[\frac{1}{r}\frac{\partial(rA_\theta)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right]\hat{\mathrm{e}}_\varphi$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathrm{e}}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\mathrm{e}}_\theta + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial f}{\partial \varphi}\hat{\mathrm{e}}_\varphi$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\,\mathrm{sen}\,\theta}\frac{\partial}{\partial \theta}\left(\mathrm{sen}\,\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\,\mathrm{sen}^2\,\theta}\frac{\partial^2 f}{\partial \varphi^2}$$

10.8.4 Teoremas do Cálculo Vetorial

$$\oint \mathbf{A}\cdot d\mathbf{S} = \int (\nabla\cdot\mathbf{A})\, dV \qquad \oint \mathbf{A}\cdot d\vec{\ell} = \int (\nabla\times\mathbf{A})\cdot d\mathbf{S}$$